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ABSTRACT

A finite difference solution for reverse osmosis in turbulent flow is presented. The case of a flat duct made of two parallel semi-permeable membranes is considered. The pertinent diffusion equation with the eddy diffusivity based on Deissler's expression is solved using a two-step linearized finite difference scheme, taking into account the nonlinear effect due to the varying water flux across membrane along the axial direction. Numerical results relating concentration polarization, water flux, Sherwood number water recovery ratio and operating conditions are given for a variety of cases.

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INTRODUCTION

A number of investigations have been reported in the literature in recent years relating the performance of reverse osmosis systems with system configurations and operating variables. The earlier work of Merten (7) described the concentration polarization phenomenon in reverse osmosis operation based on one-dimensional diffusion and film model. A more exact analysis was performed by Sherwood, Brian, Fisher and Dresner (9). Based on Berman's expression for the velocity profile, the two-dimensional diffusion equation was solved in a manner similar to that of the classical Graetz-Nusselt problem assuming constant longitudinal water flux across the membrane. It was pointed out in a subsequent study (5) that this rather arbitrary assumption of constant water flux can be removed and a perturbation solution valid for short axial length was obtained. A finite difference solution for the same problem has also been carried out (1). For a more general two-dimensional case, Gill, Tien and Zeh (6) obtained an asymptotic solution to the diffusion equation, in which the velocity profile within the concentration boundary layer was approximated by a linear expression because of the relatively high value of the Schmidt number of the water-salt system. The approximate method of integral solution has also been found useful for the solution of reverse osmosis problems and studies on the simultaneous development of velocity and concentration profiles for a number of cases has already been made (10,11). Although different assumptions have been made in these studies, the flow conditions of the works cited above (5,6,9,10,11) were assumed to be laminar.

The assumption of laminar flow for reverse osmosis process is well understood since almost all of these earlier studies were concerned primarily with desalination of sea water. For sea water with an average salt concentration of 3.5%, the osmotic pressure is around 360 psi. In terms of the membranes available today as well as those anticipated in the near future, the operating pressure for a reverse osmosis system should be about 1000 psi in order to have reasonable production capacity. In view of this relatively high operating pressure and also owing to the fact that the distance between adjacent membranes is extremely narrow, the flow within a membrane duct would necessarily be laminar.

The same consideration, however, does not remain valid if fairly dilute systems are to be treated such as the treatment and renovation of waste water. In turbulent flow, mass transfer rates are usually enhanced by the added eddy diffusion. Consequently, concentration polarization in the turbulent flow case would be somewhat less severe and so does its distribution effect. Turbulent flow in reverse osmosis may very well be more advantageous even with added pressure drops.

For reverse osmosis in turbulent flow, Dresner (4) obtained the following expression:

$$\Gamma = c_w^+ - 1 = 25 \left(\frac{v_w}{u_B}\right) \left[N_{Re_E}\right]^{1/4} \left[N_{Sc}\right]^{2/3} \qquad \text{(I-1)}$$

Similar expressions have also been given by Sherwood and co-workers (8). The above expression, undoubtedly valuable in a qualitative sense, does not yield precise information required for a better understanding of the process because of the substantial difference between the basic assumptions used and the actual operating conditions. Consequently, great caution must be exercised for the proper interpretation of the results of these expressions. For example, an average value of v_w needs to be defined since in practice v_w does not remain constant but decreases along the axial direction.

The object of this investigation is to present a detailed analysis on the mass transfer characteristics of reverse osmosis in turbulent flow. Specifically, the system considered is that of parallel membrane sheets with salt solution flowing in between. A finite difference solution for the pertinent diffusion equation using Deissler's expression for eddy diffusivity is obtained using a two-step linearized finite difference scheme. The use of finite difference is primarily due to the fact that comparatively fewer assumptions are involved in this method. Furthermore, the finite difference method is applicable to both large and small longitudinal distances and, therefore, less restrictive as compared with the series solution employed for the laminar case.

FORMULATION OF PROBLEM

A schematic description of the physical problem is given in Figure 1. Salt solution with an initial concentration c_I and osmotic pressure $(P_{os})_I$ under pressure is forced through a flat duct whose walls are made of parallel semi-permeable membranes. The distances between the membranes is 2b. The flow is assumed to be turbulent.

Because of symmetry, it is only necessary to consider half of the conduit. The pertinent diffusion equation, in dimensionless form, can be written as follows:

$$u^+ b^+ \frac{\partial c^+}{\partial x^+} + \left[N_{Re_w}\right]\left[N_{Re}\right] v^+ \frac{\partial c^+}{\partial z^+} = \left[N_{Re_I}\right] \frac{\partial}{\partial z^+}\left(A \frac{\partial c^+}{\partial z^+}\right) \qquad \text{(II-1)}$$

where

$$A = \frac{1}{N_{Sc}} + \frac{\varepsilon}{\nu} \qquad \text{(II-2)}$$

$$N_{Re_w} = \frac{bv_{w_o}}{\nu} \qquad \text{(II-3a)}$$

$$N_{Re_I} = \frac{bU_o}{\nu} \qquad \text{(II-3b)}$$

$$b^+ = \frac{b}{\nu}\sqrt{\frac{\tau_w}{\rho}} \tag{II-4}$$

The boundary conditions are:

$$c^+ = 1 \text{ at } x^+ = 0,\ 0 \leq z^+ \leq 1 \tag{II-5}$$

$$\frac{\partial c^+}{\partial z^+} = 0 \text{ at } z^+ = 1,\ x^+ > 0 \tag{II-6}$$

$$-\left(\frac{\partial c^+}{\partial z^+}\right) = \left[N_{Re_w}\right]\left[N_{Sc}\right] v^+ c^+ \text{ at } z^+ = 0,\ x^+ > 0 \tag{II-7}$$

where N_{Sc} is the Schmidt number and is taken to be 560. In Equation (II-7), it is implicitly assumed that the membrane rejects the salt completely.

The water flux, v_w, across the membrane is given by the phenomenological law

$$v_w = A\left[\Delta P - (P_{os})\right] \tag{II-8}$$

where A is the membrane constant, ΔP is the pressure drop across the membrane and P_{os} is the osmotic pressure. Assuming that the concentration variation of osmotic pressure is linear, one has,

$$v_w = A(\Delta P)\left[1 - \frac{(P_{os})_I}{\Delta P}\ \frac{c_w}{c_I}\right] \tag{II-9}$$

The dimensionless quantity, v_w^+, is then

$$v_w^+ = \frac{v_w}{v_{w_o}} = \frac{1}{1-B_2} - \frac{B_2}{1-B_2}\, c_w^+ \tag{II-10}$$

and

$$B_2 = (P_{os})_I/\Delta P \tag{II-11}$$

The fraction of water recovered, Q^+, may be obtained by integrating the wall velocity with respect to the axial distance. Q^+ is given by the following equation:

$$Q^+ = \left[N_{Re_w} \right] \int_o^{x^+} v_w^+ \, dx^+ \qquad \text{(II-12)}$$

For the solution of Equation (II-1), it is necessary to describe the velocity field as well as the eddy diffusivity which is part of the term, A. It is assumed that because the permeation velocity, v_w, is of much smaller magnitude as compared to the axial velocity, u, its effect may be ignored and the velocity expressions given by Deissler (3) may be used. These are:

$$u^+ = \int_o^{y^+} \frac{dy^+}{1 + (0.124)^2 \, u^+ y^+ \left[1 - \exp\left(- 0.124^2 \, u^+ y^+\right) \right]} \qquad \text{(II-13a)}$$

$$y^+ < 26$$

and

$$u^+ = u_1^+ + \frac{1}{0.36} \ln \left(\frac{y^+}{26} \right), \; y^+ > 26 \qquad \text{(II-13b)}$$

where u_1^+ is taken to be the value of u^+ at $y^+ = 26$. The transverse velocity component, v^+, is assumed to be independent of z^+ and equal to the value of v_w^+. The eddy diffusivity term, $\frac{\epsilon}{\nu}$, is:

$$\frac{\epsilon}{\nu} = (0.124)^2 \, u^+ y^+ \left[1 - \exp\left(- 0.124^2 \, u^+ y^+\right) \right], \; y^+ < 26 \qquad \text{(II-14a)}$$

$$\frac{\epsilon}{\nu} = 0.36 \, y^+ \left(1 - \frac{y^+}{b^+} \right), \; y^+ > 26 \qquad \text{(II-14b)}$$

Equations (II-1) together with Equations (II-13a,b) - (II-14a,b) and the boundary conditions of Equations (II-5) -- (II-7), give a complete description of the physical problem and the solution of this system of equations represents the main object of the present investigation. It should be mentioned that the parameter, b^+, present in Equation (II-1) varies along the axial direction because of the continuous permeation of pure water across the membrane. The quantity, b^+, can be related to the equivalent Reynolds number, N_{Re_E}, as follows:

$$b^+ = \frac{N_{Re_E}}{4}\sqrt{\frac{f}{2}} \qquad \text{(II-15)}$$

Furthermore, the fraction factor, f, can be expressed in terms of N_{Re_E} by the well known Blasius equation,

$$f = 0.079\left[N_{Re_E}\right]^{-1/4} \qquad \text{(II-16)}$$

Since water is being withdrawn from the system, N_{Re_E} decreases continuously. As a consequence, the parameter, b^+, the velocity profile, u^+, as well as the eddy diffusivity expressions all vary along the axial direction. The manner in which these variations are being accounted for will be described in the following section.

NUMERICAL SOLUTION

A two-step linearized finite difference scheme which has been proved successful in the past (1) is used for the solution of Equation (II-1) with the boundary conditions of Equations (II-5) - (II-7). A brief description of the procedure is given as follows.

The first step of the computation scheme consists in obtaining an approximate concentration profile for the advance longitudinal position in a marching process, i.e. to obtain concentration value at the i+1st x^+ position using the concentration values at the preceding mesh points. For this purpose, a corresponding difference equation of Equation (II-1) is obtained by approximating the various derivatives as follows:

$$\frac{\partial c^+}{\partial x^+} = \frac{c^+_{i+1,j} - c^+_{i,j}}{\Delta x^+} \qquad \text{(III-1)}$$

$$\frac{\partial c^+}{\partial z^+} = \frac{c^+_{i+1,\,j+1} - c^+_{i+1,\,j-1}}{2\Delta z^+} \qquad \text{(III-2)}$$

$$\frac{\partial}{\partial z^+}\left(\Lambda \frac{\partial c^+}{\partial z^+}\right) = \frac{\Lambda_{j+1/2}\, c^+_{i+1,\,j+1} - (\Lambda_{j+1/2} + \Lambda_{j-1/2})\, c^+_{i+1,j} + \Lambda_{j-1/2}\, c^+_{i+1,\,j-1}}{(\Delta z^+)^2} \tag{III-3}$$

The mesh network is shown in Figure 2.

Substituting Equations (III-1) - (III-3) into Equation (II-1), after some rearrangement, the following expression is obtained:

$$\left\{\frac{\left[N_{Re_w}\right]\left[N_{Re_I}\right]}{2\Delta z^+} v^+_{i,j} - \frac{\left[N_{Re_I}\right](\Lambda_{j+1/2})}{(\Delta z^+)^2}\right\} c^+_{i+1,\,j+1}$$

$$+\left\{\frac{u^+_{i,j}\, b^+_{i,j}}{\Delta x^+} + \frac{\left[N_{Re_I}\right](\Lambda_{j+1/2} + \Lambda_{j-1/2})}{(\Delta z^+)^2}\right\} c^+_{i+1,j}$$

$$-\left\{\frac{\left[N_{Re_w}\right]\left[N_{Re_I}\right]}{2\Delta z^+} + \frac{\left[N_{Re_I}\right](\Lambda_{j-1/2})}{(\Delta z^+)^2}\right\} c^+_{i+1,\,j-1}$$

$$= \frac{u^+_{i,j}\, b^+_{i,j}}{\Delta x^+} c^+_{i,j} \tag{III-4}$$

Equation (III-4) for j=2,3 .... n+1, yields n equations with n+2 unknowns, i.e. $c^+_{i+1,j}$, j=1,2 ... n+1, n+2 (j=1 corresponds to the membrane wall and j=n+1 corresponds to the axis of the channel). However, the boundary condition of Equation (II-6) yields $c^+_{i+1,\,n+2} = c^+_{i+1,\,n}$. Also, from Equation (II-7), one has,

$$\left\{\frac{1}{\Delta z^+} - \left[N_{Re_w}\right]\left[N_{Sc}\right] v^+_{i+1,1}\right\} c^+_{i+1,1} - \frac{1}{\Delta z^+} c^+_{i+1,2} = 0 \tag{III-5}$$

Here, $v^+_{i+1,1}$ is approximated by $v^+_{i,1}$. Thus, one now has n+1 equations for the n+1 unknown concentrations in the advance x^+ position. The system of linear tridiagonal equations is solved by Thomas method (2). This method is equivalent

to plain Gaussian elimination, but is known to avoid the error growth associated with the back solution of the elimination method. The first approximations for $c^+_{i+1,1}$ through $c^+_{i+1,n+1}$ are thus obtained. Knowing $c^+_{i+1,1}$, $\overline{v^+_{i+1,1}}$ may be obtained from Equation (II-10). The bar over $v^+_{i+1,1}$ indicates that the value is only the first approximation.

The second step involves the formulation of the Crank Nicholson equation for Equation (II-1). This, after some rearrangement, is given as:

$$\left\{ \phi\varepsilon^* - \frac{\left[N_{Re_I}\right] \varepsilon^* (A_{j+1/2})}{(\Delta z^+)^2} \right\} c^+_{i+1,\, j+1}$$

$$+ \left\{ \frac{b^+_{i,j}\, u^+_{i,j}}{\Delta x^+} + \frac{\left[N_{Re_I}\right] \varepsilon^* (A_{j+1/2} + A_{j-1/2})}{(\Delta z^+)^2} \right\} c^+_{i+1,j}$$

$$- \left\{ \phi\varepsilon^* + \frac{\left[N_{Re_I}\right] \varepsilon^* (A_{j-1/2})}{(\Delta z^+)^2} \right\} c^+_{i+1,\, j-1}$$

$$= \frac{b^+_{i,j}\, u^+_{i,j}\, c^+_{i,j}}{\Delta x^+} - \phi G + \frac{H A_j \left[N_{Re_I}\right]}{(\Delta z^+)^2} \qquad \text{(III-6)}$$

where,

$$\phi = \frac{\left[N_{Re_w}\right]\left[N_{Re_I}\right]}{2(\Delta z^+)} \left[\varepsilon^* \overline{v^+_{i+1,1}} + (1-\varepsilon^*)\, v^+_{x,1} \right] \qquad \text{(III-7)}$$

$$G = (1-\varepsilon^*) \left[c^+_{i,j+1} - c^+_{i,j-1} \right] \qquad \text{(III-8)}$$

$$H = \frac{1-\varepsilon^*}{A_j} \left[A_{j+1/2}\, c^+_{i,j+1} - (A_{j+1/2} + A_{j-1/2})\, c^+_{i,j} + A_{j-1/2}\, c^+_{i,j-1} \right] \qquad \text{(III-9)}$$

and ε^* is the weighting coefficient.

For the present case, a value of $\epsilon^* = 0.5$ corresponding to Crank Nicholson weighting is used. Similar to the previous step, together with the boundary conditions of Equation (II-6) and Equation (III-5), Equation (III-6) defines n+1 equations with n+1 unknowns, i.e. $c_{i+1,j}$, j=1,2 ... n+1. The value of $v_{i+1,1}$ which appears in Equation (III-5) is approximated by the arithmetic average of $\overline{v^+_{i+1,1}}$ and $v^+_{i,1}$. This system of n+1 equations in the unknowns, $c^+_{i+1,1}$ through $c^+_{i+1,\,n+1}$, is then solved and the new concentration values are used to obtain a second approximation of $\overline{v^+_{i+1,1}}$, which is then used for another round of calculations. This iteration procedure may be carried out as many times as required, although in almost all cases, three iterations were found adequate. The concentration values from the last iteration performed are taken to be the correct values. This, in turn, are the values used in the calculations for the next advance x^+ position.

The variation in N_{Re_E} Reynolds number based on equivalent diameter, is given by,

$$\left[N_{Re_E}\right]_{x^+} = \left[N_{Re_E}\right]_I (1-Q^+) \qquad \text{(III-10)}$$

The subscript I refers to the value at the inlet. Q^+ is obtained from Equation (II-12). Using Equations (II-15) and (II-16), it can be shown that,

$$(b^+)_{x^+} = b^+_I (1-Q^+)^{7/8} \qquad \text{(III-11)}$$

b^+ is calculated for each step and the current value of b^+ is used in the computation scheme. As pointed out earlier, the velocity and eddy diffusivity expressions involve b^+ and hence need to be recalculated as b^+ varies along the longitudinal direction. In the computation performed in this work, these values were recalculated for every five percent increase in the water recovery instead of after every step. This is believed to be a reasonable balance between computer time involved in recalculating the quantities and adherence to the physical model.

RESULTS AND DISCUSSION

The numerical computations were performed on an IBM System/360 digital computer. The following combination of parameters was considered.

$$b_I^+ = 100, 200, 300$$

(These correspond to inlet Reynolds numbers $\left[N_{Re_E}\right]_I$ of 5967, 13175 and 20941, respectively.)

$$N_{Re_w} = 0.006, 0.015$$

$$B_2 = 0, 1/4, 1/3, 1/2$$

($B_2 = 0$ corresponds to very dilute solutions with high operating pressures.) For each set of specified value of b_I^+, N_{Re_w} and B_2, the quantities c_w^+, v_w^+, Q^+, b^+ and N_{Re_E} were calculated as functions of x^+. The results for c_w^+ vs x^+ are presented graphically in Figures 3 to 8. Figures 9 and 10 show typical results for v_w^+ and Q^+ as functions of x^+ for different values of B_2 for $b_I^+ = 200$ and $N_{Re_w} = 0.015$.

It was stated in the previous section that concentration polarization is expected to be reduced with the increased eddy diffusion. The extent of reduction of salt buildup due to the increase of turbulence (in terms of inlet Reynolds number) is clearly indicated in Figures 11 and 12 in which values of c_w^+ were plotted against x^+ with specified values of N_{Re_w} and B_2 for different values of $\left[N_{Re_E}\right]_I$. Because of the presence of the Reynolds number, in the definition of x^+, all the abscissa in these figures were referred to a common value of $\left[N_{Re_E}\right]_I$. The significant reduction of c_w with the increase of inlet Reynolds number becomes quite obvious. Furthermore, the decrease becomes more pronounced with the decrease of the value of B_2 as well as the decrease of N_{Re_w}.

As an example, at $x^+ = 20$ (based on $b_I^+ = 300$), the reduction of the excess salt concentration buildup (defined as $c_w^+ - 1$) is approximately 53% with a 3.5 fold increase of inlet Reynolds number (from 5967 to 20941) for the case of $N_{Re_w} = 0.015$ and $B_2 = 0.5$. The corresponding reduction for the case of $N_{Re_w} = 0.006$ and $B_2 = 0.25$, however, was found to be 82%.

The effect of the parameters B_2 and N_{Re_w} on the production capacity of the reverse osmosis system is indicated in Tables I and II. Based on the definitions given in the nomenclature, it can be seen that the actual amount of pure water produced in proportional to the product of Q^+ and $\left[N_{Re_E}\right]_I$, which was found to be increasing with the increase of $\left[N_{Re_E}\right]_I$. Also, this increase is most pronounced for lower values of B_2 and higher values of N_{Re_w}, which, physically speaking, corresponds to the case of very dilute feed stream operating at relatively high pressures with high capacity membranes. It is interesting to note that these conditions are most likely to be met if reverse osmosis is to be developed as a practical means for treatment of waste water.

The results of this work are also compared with those based on the one-dimensional model. The one-dimensional diffusion equation based on complete salt rejection at the surface of the membrane can be written as:

$$v_w c + (D_s + \varepsilon) \frac{dc}{dy} = 0 \qquad \text{(IV-1)}$$

Using Deissler's expression for eddy diffusivity [Equations (II-14a) and (II-14b)], the following result is obtained after simple integration,

$$\frac{c_w}{c_B} = \exp\left\{ \frac{8.96\, v_w \left[N_{Sc}\right]^{3/4}}{u_B} \sqrt{\frac{2}{f}} \right\} \qquad \text{(IV-2)}$$

A mass balance on the salt indicates that,

$$c_B^+ = \frac{c_B}{c_I} = \frac{1}{1-Q^+} \qquad \text{(IV-3)}$$

In terms of the dimensionless quantities defined in the present work, Equation (IV-2) may be written as,

$$c_w^+ = \exp\left\{ \frac{(8.96)\left[N_{Re_w}\right]\left[N_{Sc}\right]^{3/4}}{b_I^+} \frac{(\alpha-\beta\, c_w^+)}{(1-Q^+)^{7/8}} - \ln(1-Q^+) \right\} \qquad \text{(IV-4)}$$

where,

$$\alpha = \frac{1}{1-B_2} \qquad \text{(IV-5a)}$$

$$\beta = \alpha B_2 \qquad \text{(IV-5b)}$$

Equation (IV-4) defines the relationship between Q^+ and c_w^+ for a given combination of N_{Re_w}, b_I^+ and B_2. In order to obtain the numerical results of c_w^+ vs Q^+, a value of Q^+ is specified and the resulting implicit equation in c_w^+ can be solved using Newton Raphson iteration scheme. Comparison between the results of the finite difference solution and those of Equation (IV-4) are given in Figures 13 and 14 for two cases; $b_I^+ = 200$, $N_{Re_w} = 0.006$, $B_2 = 0.5$, and $b_I^+ = 150$, $N_{Re_w} = 0.015$ and $B_2 = 0.5$. In terms of excess salt buildup (i.e. $c_w^+ - 1$), the one-dimensional model yields much lower values and tends to underestimate the extent of concentration polarization. Furthermore, this underestimation increases as one proceeds downstream.

A conventional way of describing the mass transfer characteristic is in terms of the mass transfer coefficient which can be defined as follows:

$$k_x = - \frac{D_s \left(\frac{\partial c}{\partial y}\right)_{y=0}}{c_w - c_b} \qquad \text{(IV-6)}$$

Accordingly, the Sherwood number, N_{Sh}, becomes

$$N_{Sh} = \frac{k_x D_s}{b} = - \left(\frac{\partial c^+}{\partial y^+}\right)_{y^+=0} / (c_w^+ - c_b^+) \qquad \text{(IV-7)}$$

Substituting Equations (II-7) and (IV-3) into Equation (IV-7), one has,

$$N_{Sh} = \frac{k_x D_s}{b} = \frac{\left[N_{Re_w}\right]\left[N_{Sc}\right](v_w^+)(c_w^+)(1-Q^+)}{(1-Q^+)\,c_w^+ - 1} \qquad \text{(IV-8)}$$

Numerical values of the local Sherwood number as a function of x^+ are shown in Figures 15a and 15b for a variety of cases. For small longitudinal distances ($x^+ \leq 15$), N_{Sh} was found to be essentially independent of B_2. For $x^+ > 20$, N_{Sh} decreases with the increase of B_2. However, the effect to B_2 on N_{Sh} is much less as compared with that on other pertinent parameters (for example, compare Figure 3 with Figure 15).

For a numerical treatment such as given in this work, the question of predicting the results for conditions not specifically covered in the numerical calculation always presents some difficulty because of the absence of the closed-form expression. Owing to the large number parameters involved in this work (i.e., N_{Re_w}, b_I^+, B_2 and N_{Sc}), only limited cases can be considered. The prediction of all pertinent quantities by direct interpolation, therefore, becomes difficult. On the other hand, the most important quantity from a practical point of view is that of the wall concentration, c_w^+. It is believed that a reasonable procedure for the prediction of c_w^+ is through the use of expression for N_{Sh} given by Equation (IV-8). Combining this expression with Equations (II-10) and (II-12) yields,

$$N_{Sh} = \frac{\left[N_{Re_w}\right]\left[N_{Sc}\right]}{(1-B_2)}\, c_w^+ (1-B_2\, c_w^+)\, \frac{(1-B_2) - N_{Re_w} \int_o^{x^+} (1-B_2\, c_w^+)\, dx^+}{(1-B_2)(c_w^+-1) - c_w^+ \left[N_{Re_w}\right] \int_o^{x^+} (1-B_2\, c_w^+)\, dx^+}$$

For a given set of parameters, the local Sherwood number as a function of x^+ can be estimated by interpolation based on the data given in Figures 15a,b. Once this information becomes available, the relationship of c_w^+ vs x^+ can be obtained

from Equation (IV-9). The main advantage of this procedure lies in the fact of the less dependency of N_{Sh} on the parameters, b_I^+, N_{Re_E} and B_2 as discussed previously. Consequently, one can predict values of N_{Sh} by interpolation with more ease. It should also be noted that in the present work, the Schmidt number is taken as 560 (i.e. NaCl-H_2O System). However, if the well-established relationship of N_{Sh} being proportional to the cubic root of N_{Sc} is assumed to be also valid for the present problem, the performance of a reverse osmosis system in turbulent flow with systems other than NaCl-H_2O can be easily predicted by this procedure.

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NOMENCLATURE

A = membrane constant

B_2 = $(P_{os})_I/\Delta P$

b = half the distance between membranes

b^+ = $\frac{b}{\nu}\sqrt{\frac{\tau_w}{\rho}}$

b_I^+ = value of b^+ at inlet

c_B = bulk concentration

c_B^+ = c_B/c_I

c_I = salt concentration at inlet

c = salt concentration

c^+ = c/c_I

c_w = wall concentration

c_w^+ = c_w/c_I

D = salt diffusion coefficient

f = friction factor

G = defined by Equation (III-8)

H = defined by Equation (III-9)

i = position index (x^+ direction)

j = position index (x^+ direction)

k_x = defined by Equation (IV-6)

N_{Re} = bu_B/ν

N_{Re_E} = $4\ N_{Re}$, Reynolds number based on equivalent diameter

N_{Re_I} = N_{Re} at inlet = bU_o/ν

NOMENCLATURE (Cont'd.)

$\left[N_{Re_E}\right]_I$ = N_{Re_E} at inlet

N_{Re_w} = defined as $\frac{bv_{w_o}}{\nu}$

N_{Sc} = Schmidt number, $\frac{\nu}{D}$

P_{os} = osmotic pressure

$(P_{os})_I$ = osmotic pressure at inlet

Q^+ = fraction of water withdrawn

u = axial component of velocity

u^+ = $u/\sqrt{\frac{\tau_w}{\rho}}$

u_1^+ = u^+ at $y^+ = 26$

u_B = bulk velocity

U_o = bulk velocity at inlet

v = transverse velocity

v^+ = v/v_{w_o}

v_w = wall velocity

v_{w_o} = wall velocity at $x^+ = 0$

x = axial coordinate

x^+ = $x/(b)\ (N_{Re_I})$

y = transverse coordinate

y^+ = $\frac{Y}{\nu}\sqrt{\frac{\tau_w}{\rho}}$

z^+ = $y^+/b^+ = y/b$

NOMENCLATURE (Cont'd.)

α = $\frac{1}{1-B_2}$

β = αB_2

Γ = defined by Equation (I-1)

ΔP = pressure drop across the membrane

$\Delta x^+, \Delta z^+$ = increments in x^+ and z^+ directions

ε = eddy diffusivity

ε^* = weighting factor (= 0.5)

Λ = $\frac{1}{N_{Sc}} + \frac{\varepsilon}{\nu}$

ν = kinematic viscosity

ρ = density

τ_w = wall shear stress

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TABLE I

$N_{Re_w} = 0.006 \quad B_2 = 0.25$

x^+ Based on $b_I^+ = 300$	$b_I^+ = 300$ $\left[N_{Re_E}\right]_I = 20941$		$b_I^+ = 200$ $\left[N_{Re_E}\right]_I = 13175$		$b_I^+ = 150$ $\left[N_{Re_E}\right]_I = 9484$		$b_I^+ = 100$ $\left[N_{Re_E}\right]_I = 5967$	
	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$
1.0	0.005927	124.12	0.00934	123.0	0.01294	122.72	0.02022	120.65
5.0	0.0294	615.24	0.0467	612.24	0.0641	607.50	0.0991	591.38
10.0	0.05826	1220	0.09174	1209	0.1246	1187	0.191	1139
15.0	0.087	1822	0.1353	1782	0.1847	1752	0.2734	1631
20.0	0.1147	2402	0.1786	2353	0.2406	2282	0.3487	2081

TABLE II

$N_{Re_w} = 0.015 \quad B_2 = 0.5$

x^+ Based on $b_I^+ = 300$	$b_I^+ = 300$ $\left[N_{Re_E}\right]_I = 20941$		$b_I^+ = 200$ $\left[N_{Re_E}\right]_I = 13175$		$b_I^+ = 150$ $\left[N_{Re_E}\right]_I = 9484$		$b_I^+ = 100$ $\left[N_{Re_E}\right]_I = 5967$	
	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$	Q^+	$\left[N_{Re_E}\right]_I (Q^+)$
1.0	0.01397	292.54	0.02139	281.76	0.0286	271.63	0.04174	249.0
5.0	0.06676	1398	0.09986	1316	0.1278	1212	0.1717	1024
10.0	0.1259	2637	0.1804	2377	0.2198	2084	0.2652	1582
15.0	0.1783	3733	0.2423	3192	0.2830	2684	0.3105	1852
20.0	0.2221	4650	0.2899	3819	0.3209	3043	0.332	1981

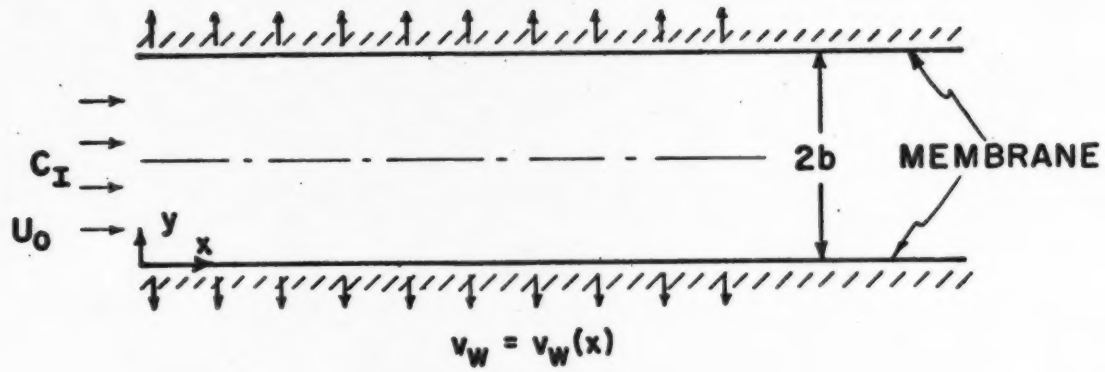

FIG. 1 SCHEMATIC DIAGRAM OF REVERSE OSMOSIS DUCT.

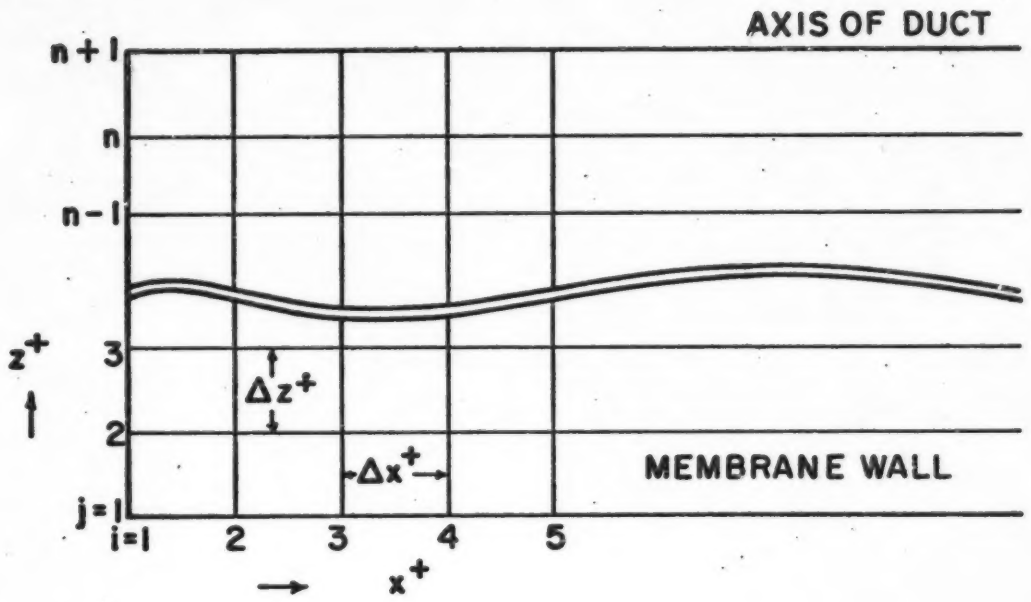

FIG. 2 MESH NET WORK.

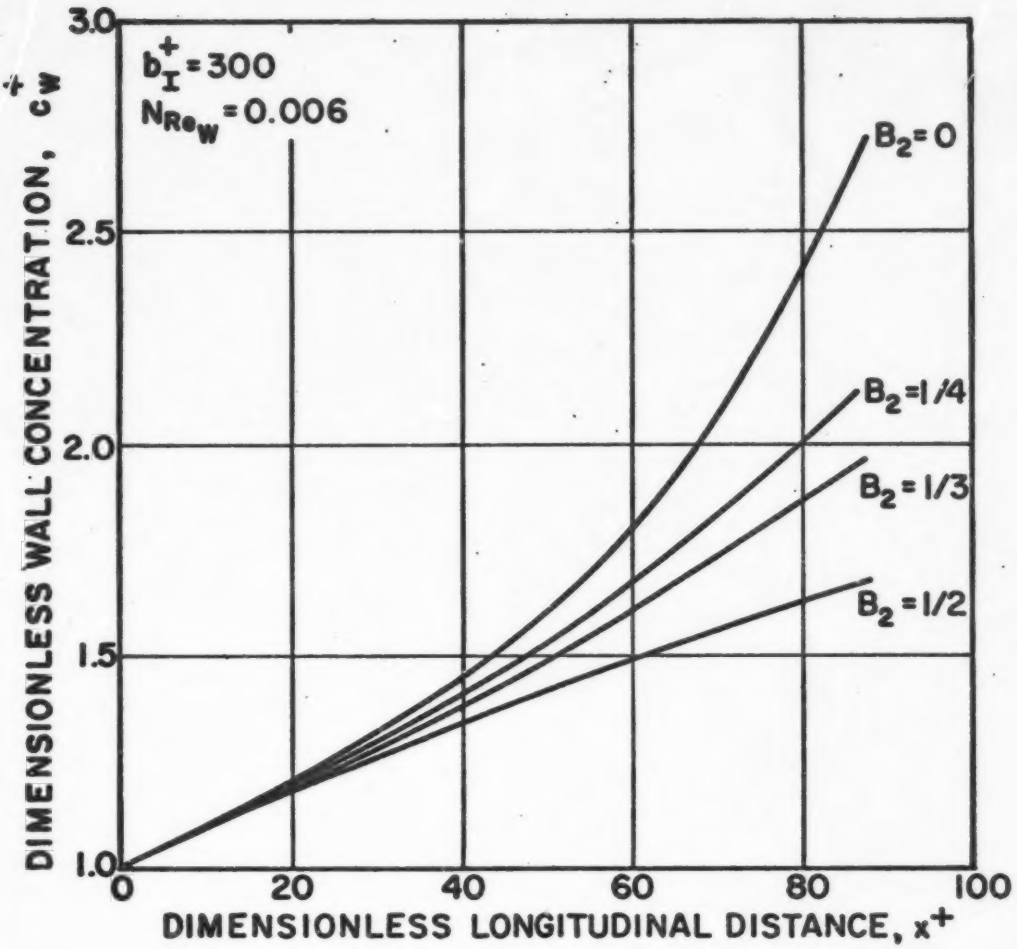

FIG. 3 DIMENSIONLESS CONCENTRATION AS A FUNCTION OF DIMENSIONLESS LONGITUDINAL DISTANCE.

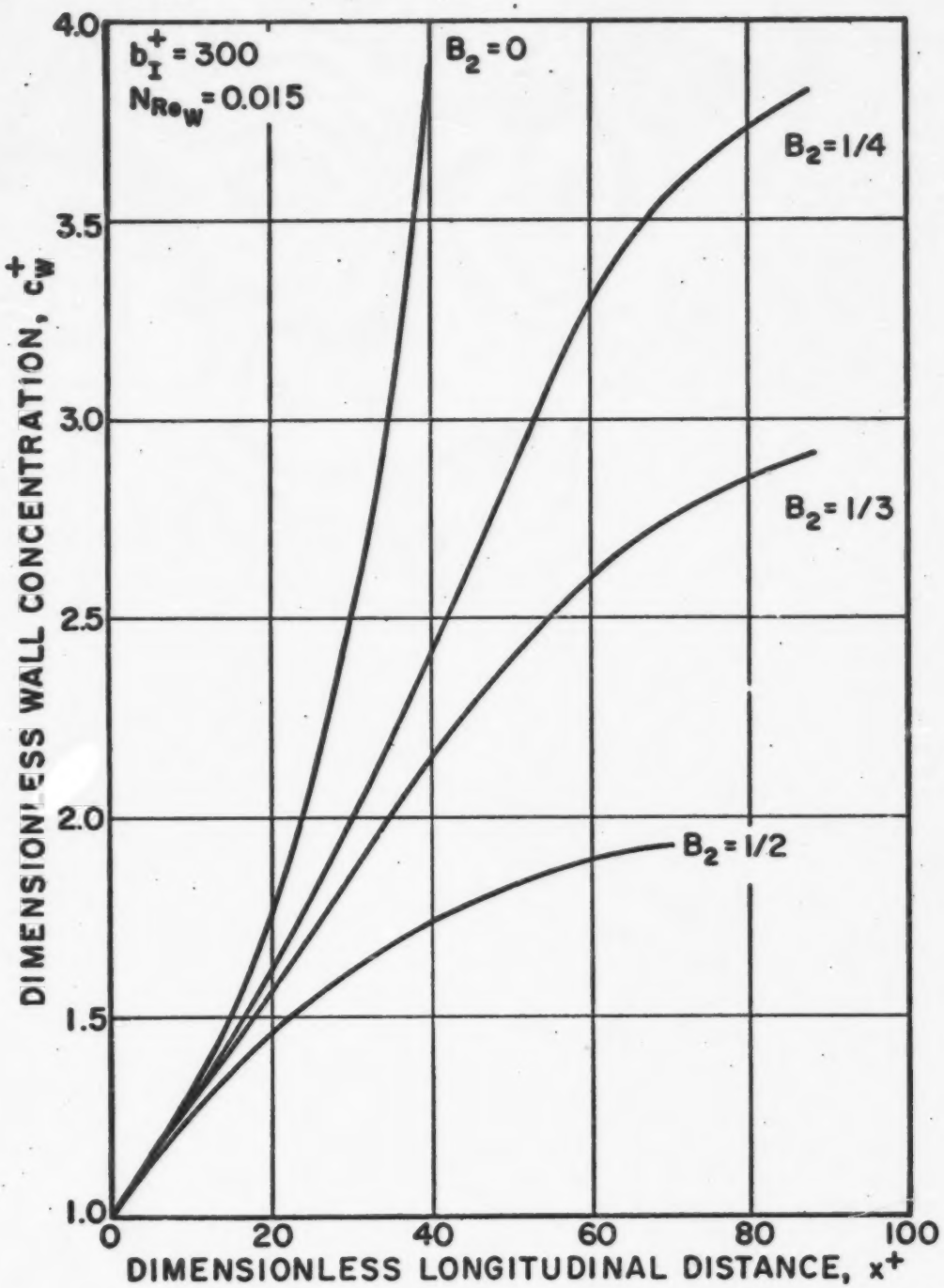

FIG. 4 DIMENSIONLESS CONCENTRATION AS A FUNCTION OF DIMENSIONLESS LONGITUDINAL DISTANCE.

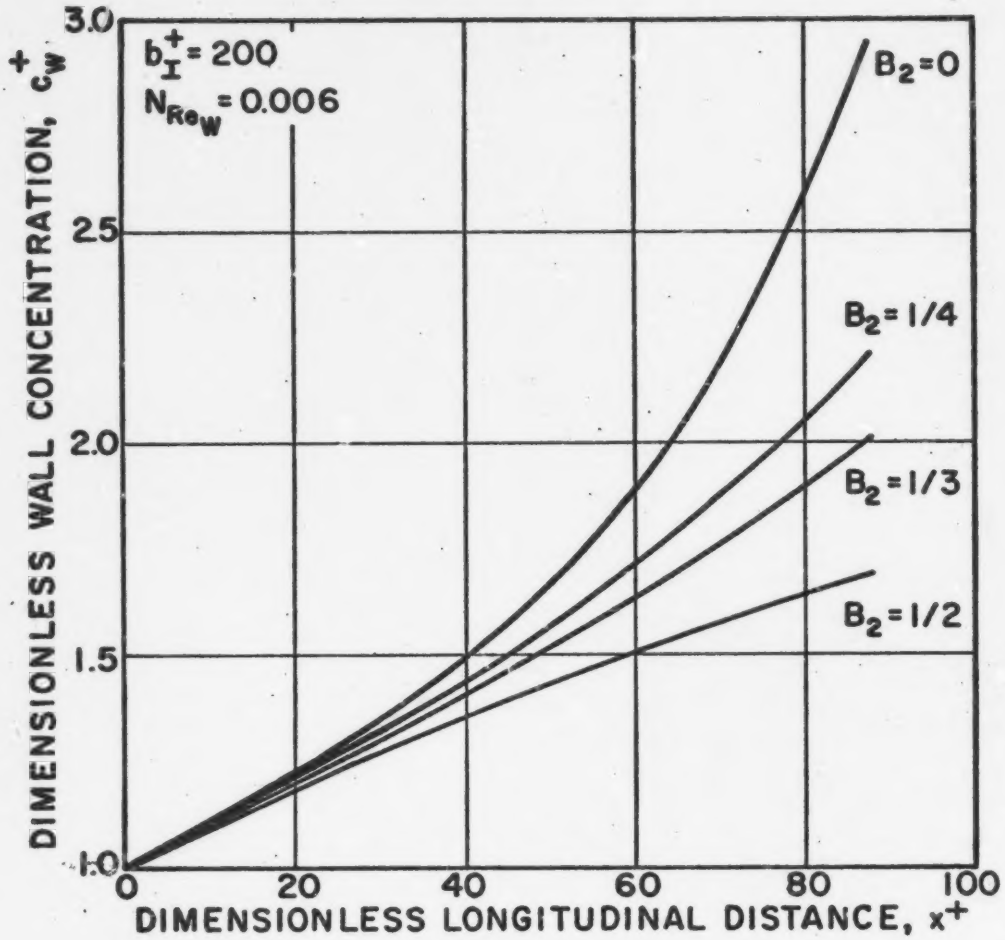

FIG. 5 DIMENSIONLESS CONCENTRATION AS A FUNCTION OF DIMENSIONLESS LONGITUDINAL DISTANCE.

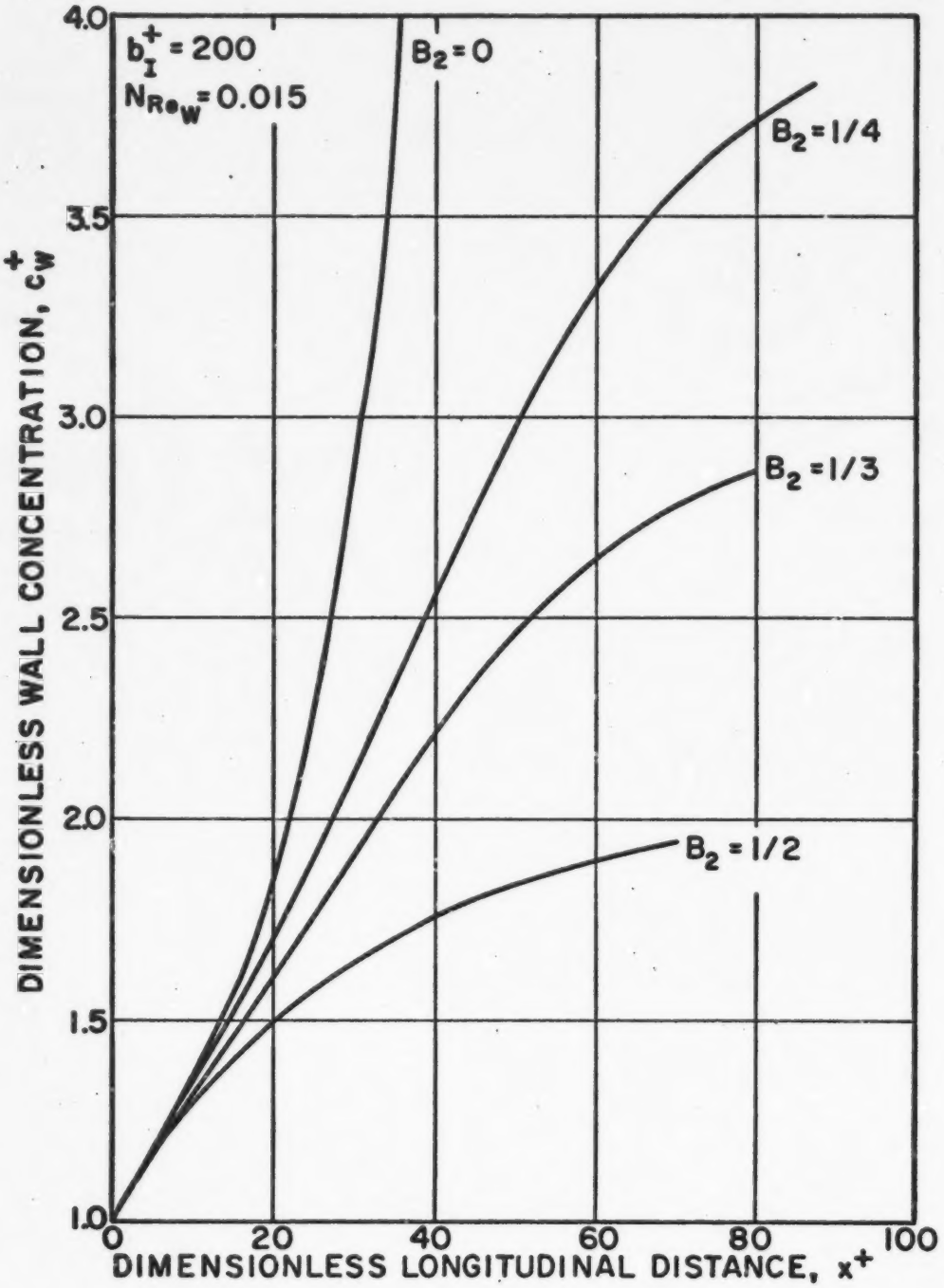

FIG. 6 DIMENSIONLESS CONCENTRATION AS A FUNCTION OF DIMENSIONLESS LONGITUDINAL DISTANCE.

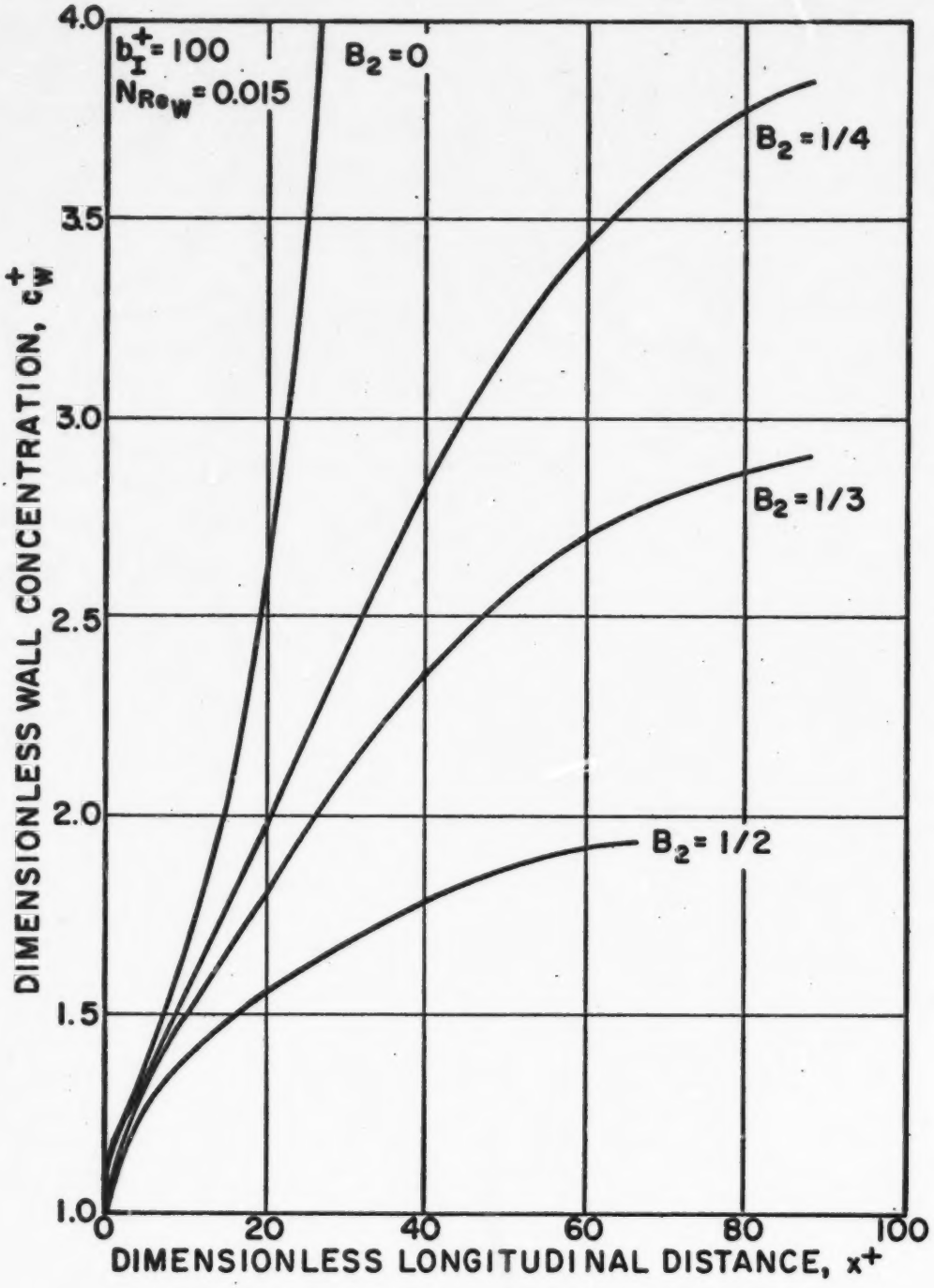

FIG. 7 DIMENSIONLESS CONCENTRATION AS A FUNCTION OF DIMENSIONLESS LONGITUDINAL DISTANCE.

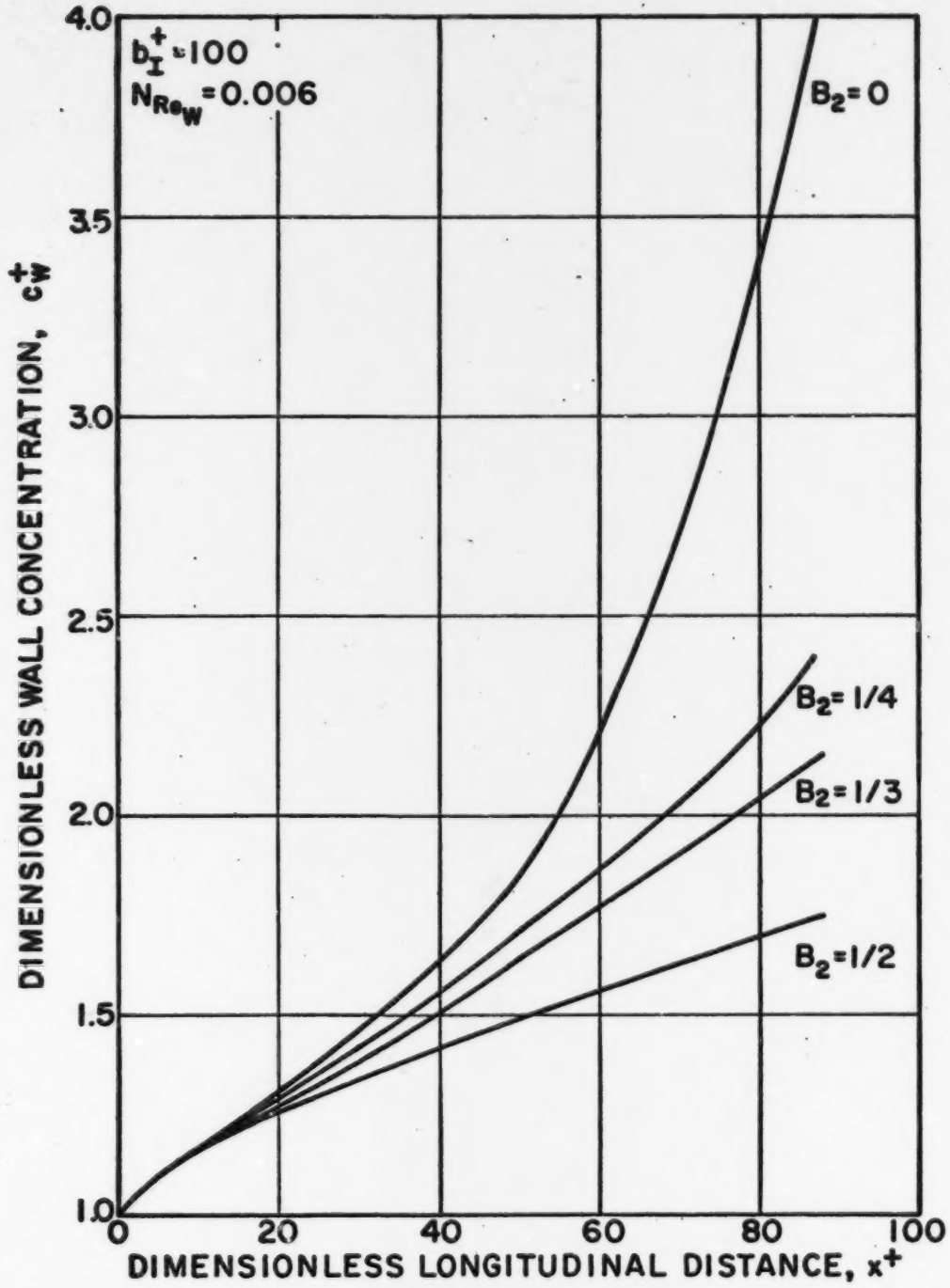

FIG. 8 DIMENSIONLESS CONCENTRATION AS A FUNCTION OF DIMENSIONLESS LONGITUDINAL DISTANCE.

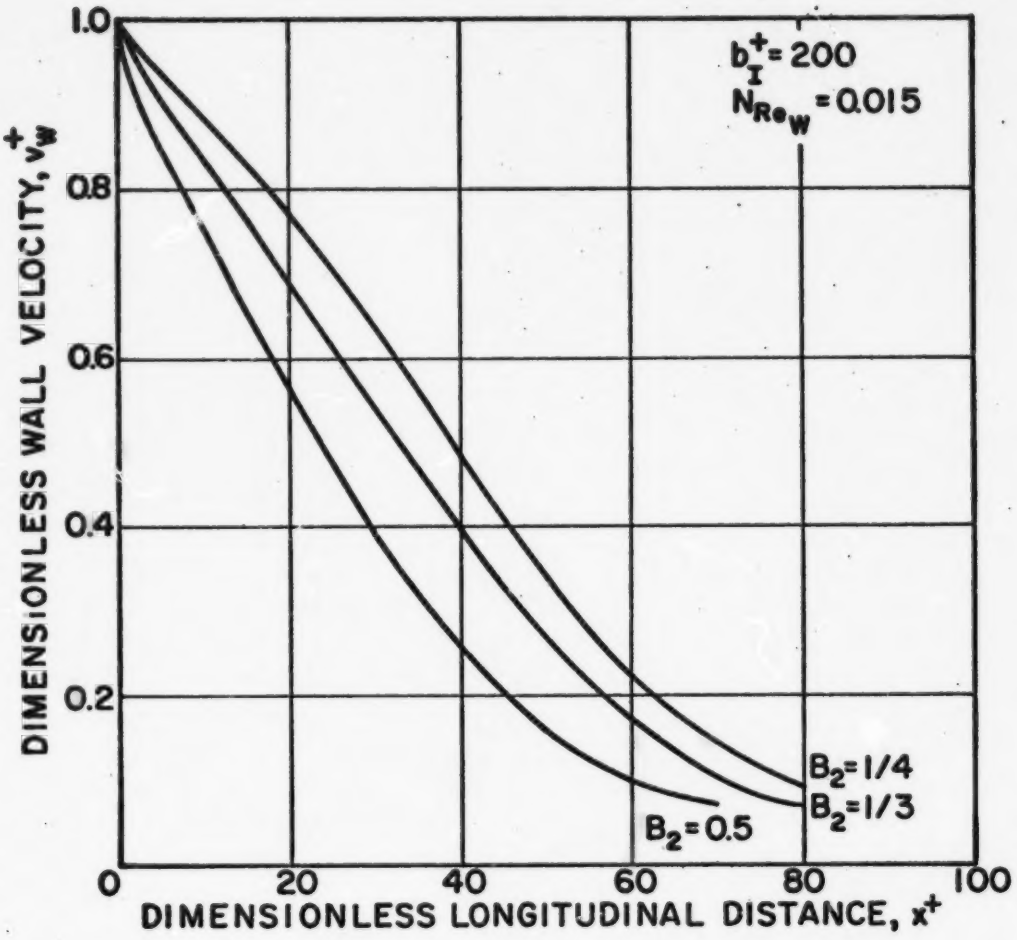

FIG. 9 DIMENSIONLESS WALL VELOCITY VS. DIMENSIONLESS AXIAL DISTANCE.

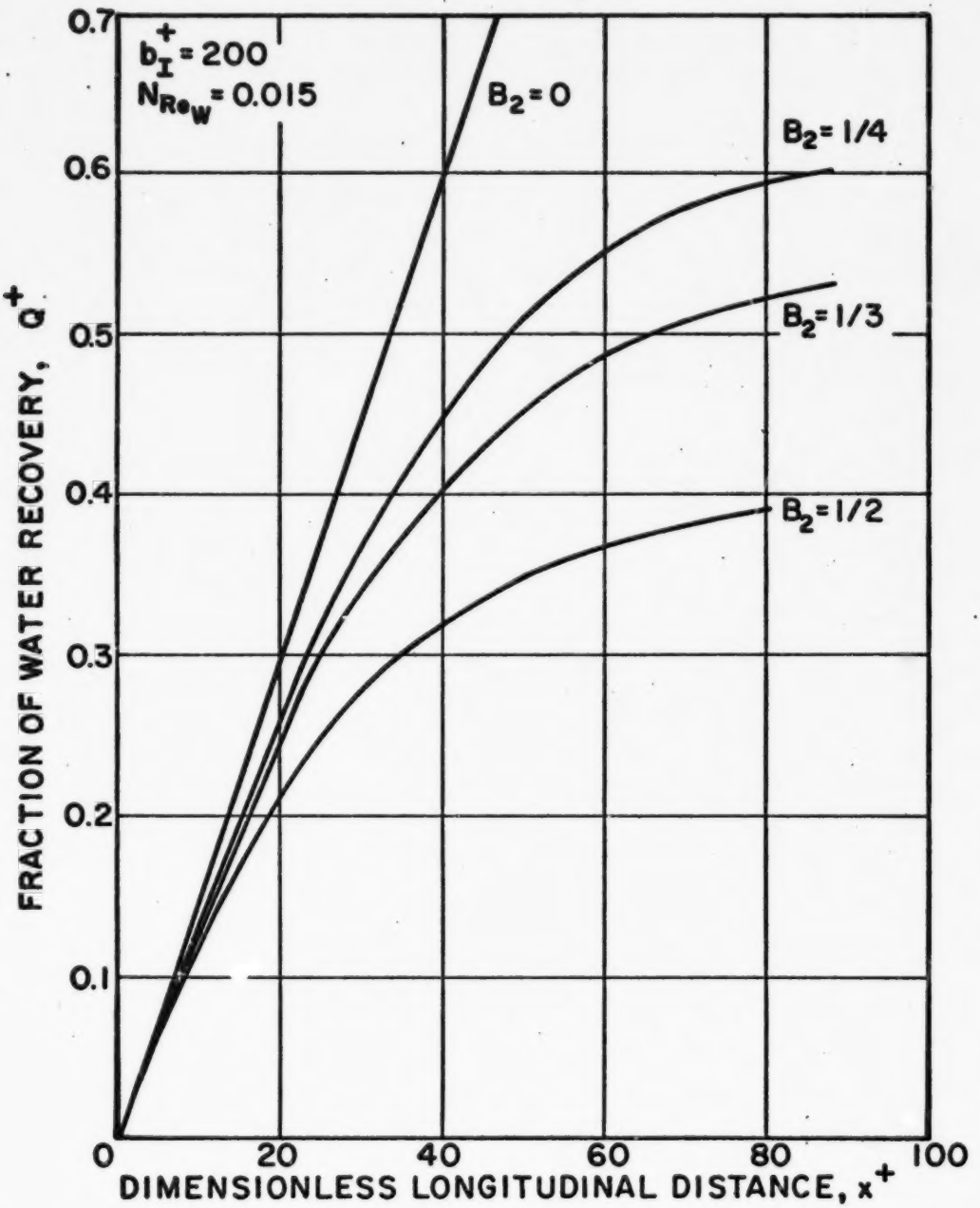

FIG. 10 FRACTION OF WATER RECOVERY VS. DIMENSIONLESS LONGITUDINAL DISTANCE.

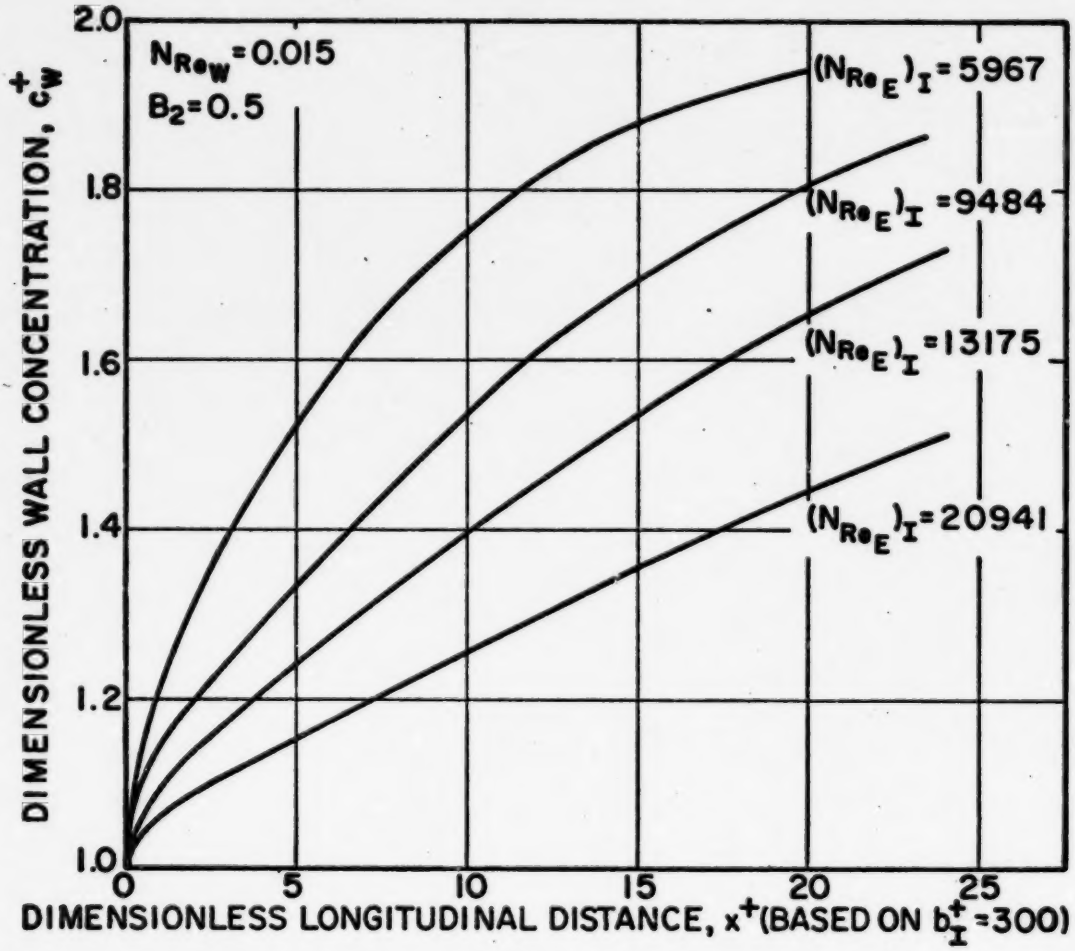

FIG. 11 EFFECT OF INLET REYNOLDS NUMBER ON CONCENTRATION POLARIZATION.

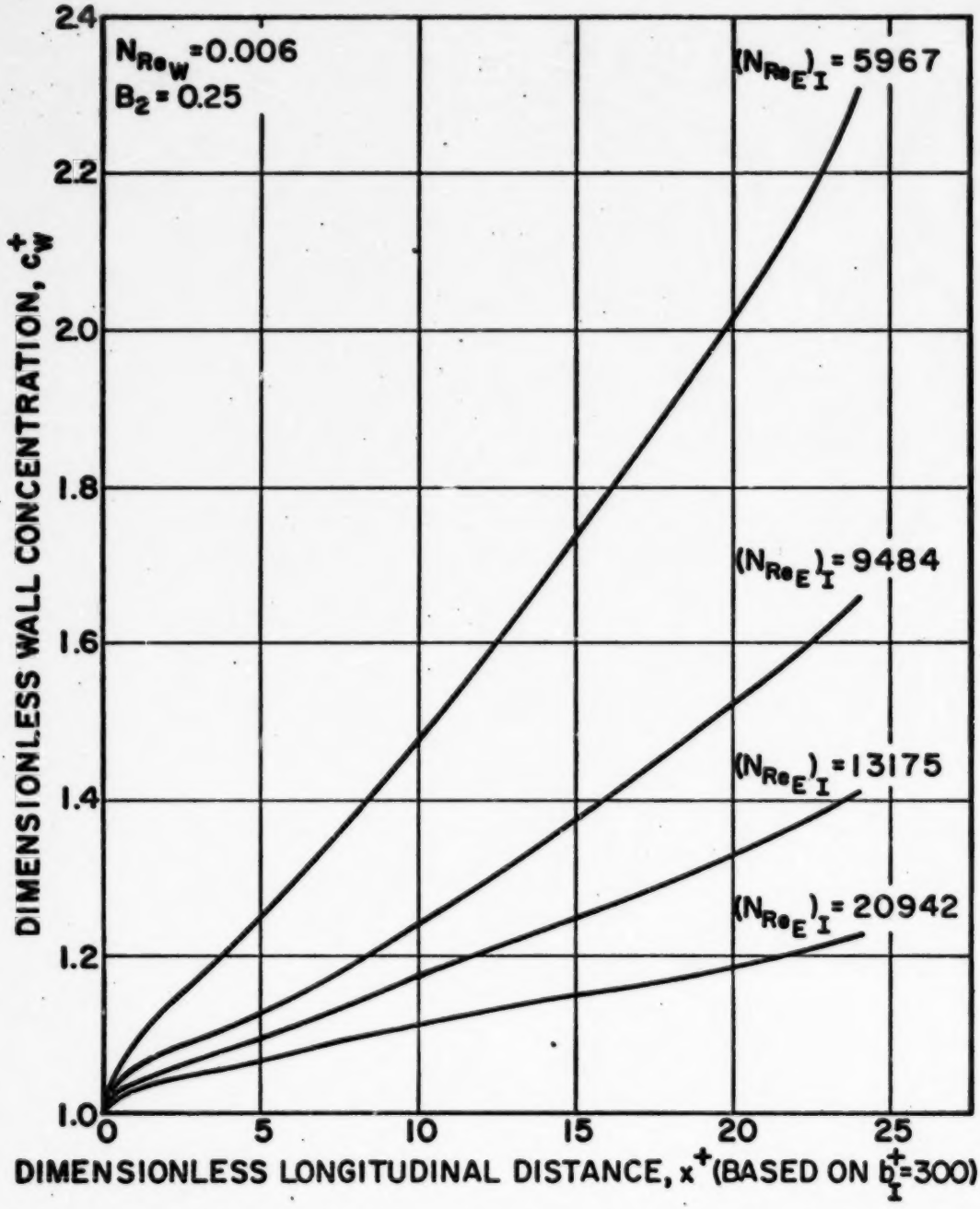

FIG. 12 EFFECT OF INLET REYNOLDS NUMBER ON CONCENTRATION POLARIZATION.

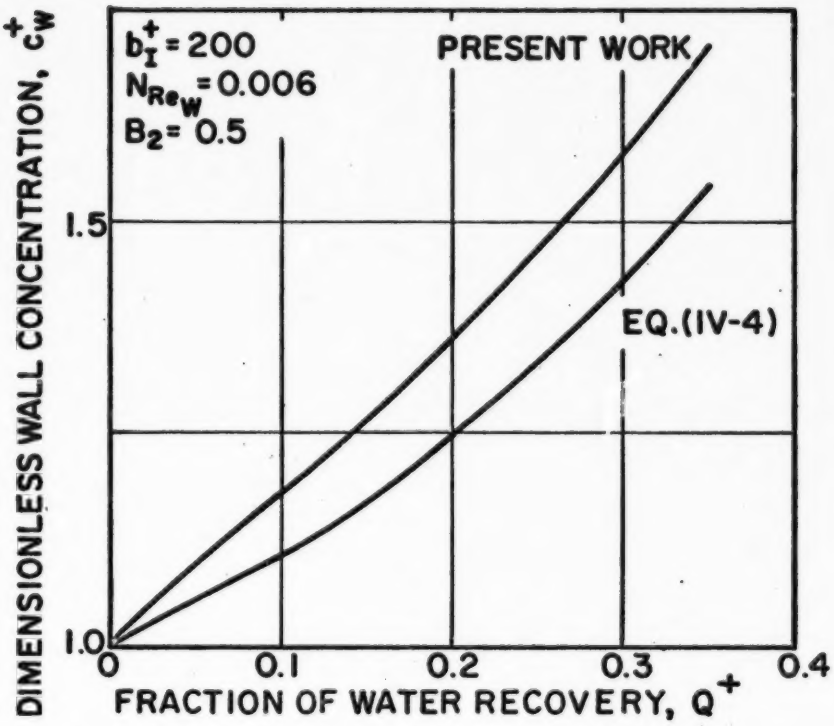

FIG. 13 COMPARISON BETWEEN PRESENT INVESTIGATION AND THAT BASED ON ONE DIMENSIONAL DIFFUSION MODEL.

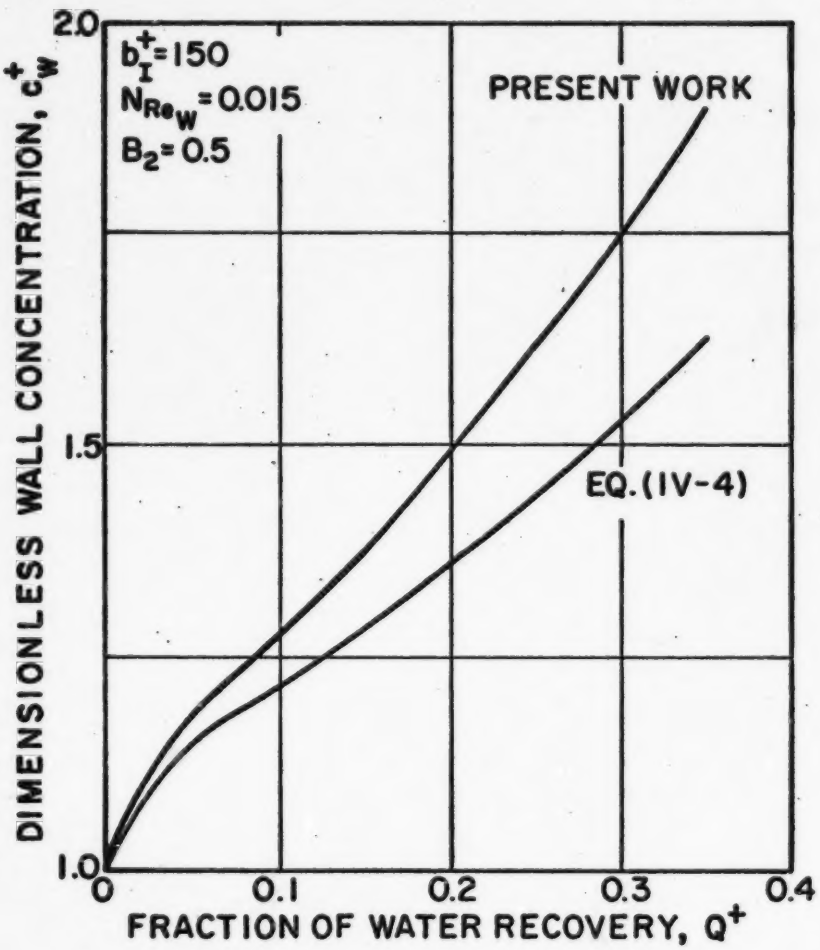

FIG. 14 COMPARISON BETWEEN PRESENT INVESTIGATION AND THAT BASED ON ONE DIMENSIONAL DIFFUSION MODEL.

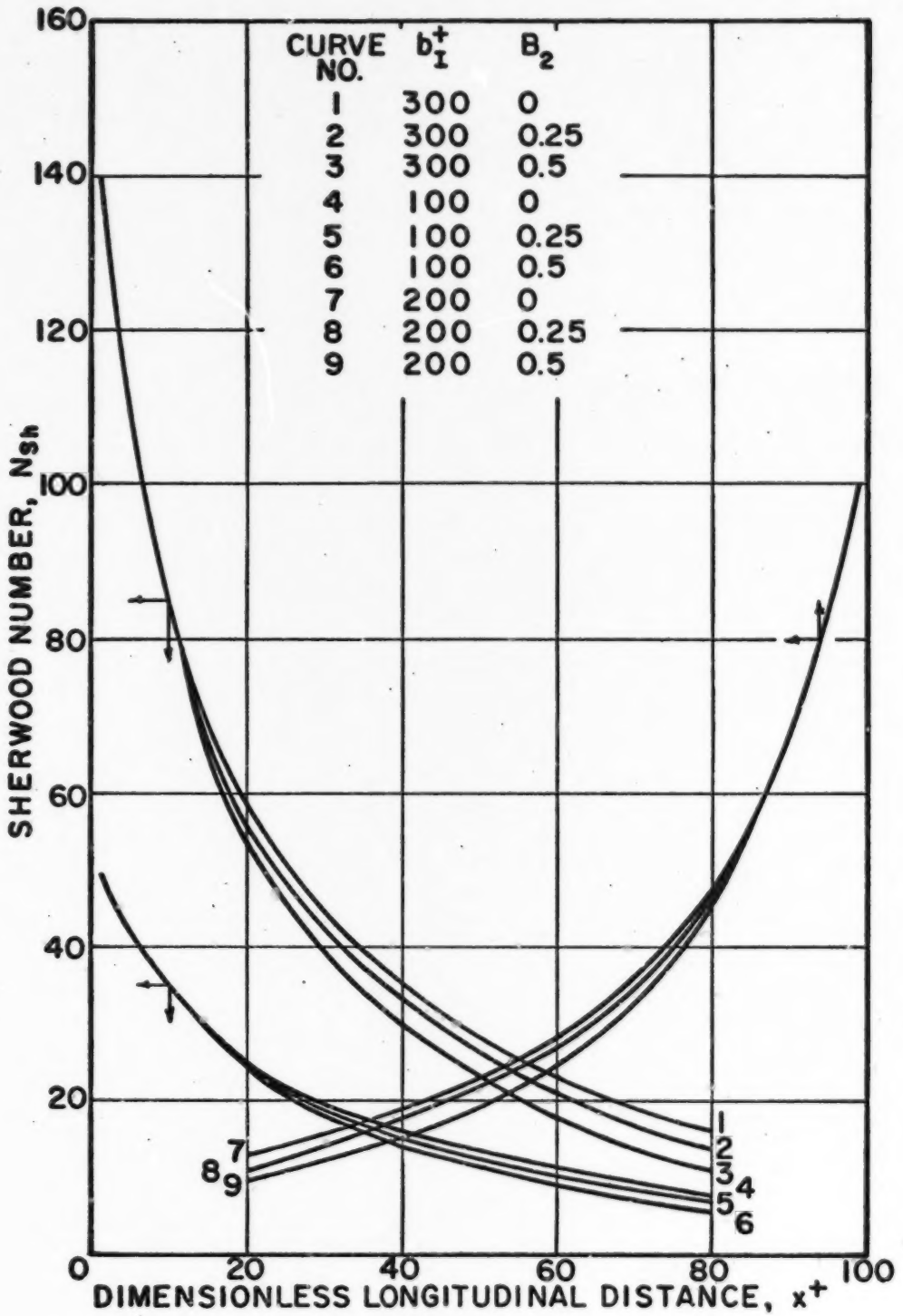

FIG. 15a SHERWOOD NUMBER VS. DIMENSIONLESS LONGITUDINAL DISTANCE.

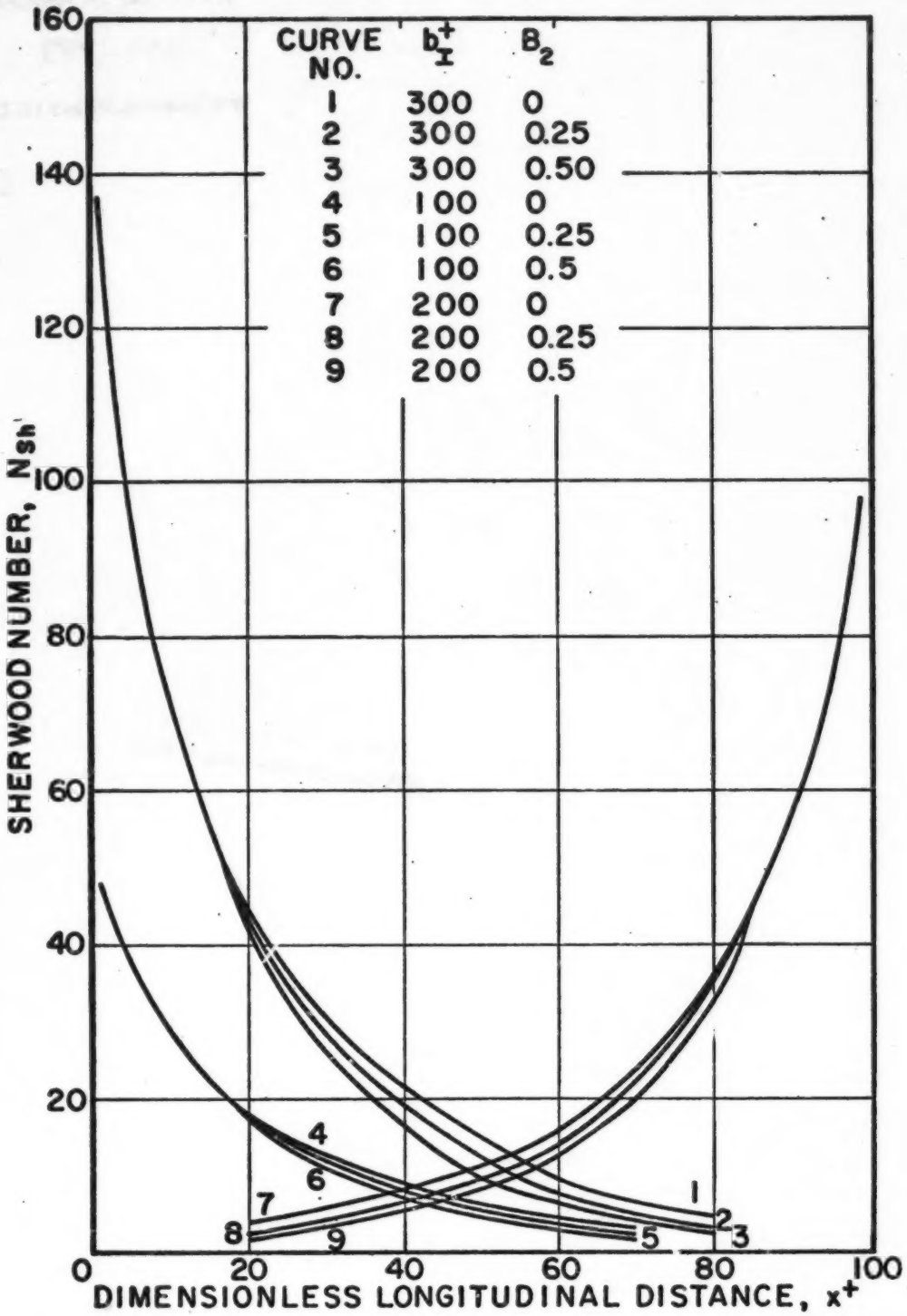

FIG. 15b SHERWOOD NUMBER VS. DIMENSIONLESS LONGITUDINAL DISTANCE.